

# The Hyperfine Splittings in Heavy-Light Mesons and Quarkonia

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Hyperfine splittings (HFS) are calculated within the Field Correlator Method, taking into account relativistic corrections. The HFS in bottomonium and the  $B_q$  ( $q=n,s$ ) mesons are shown to be in full agreement with experiment if a universal coupling  $\alpha_{HF} = 0.310$  is taken in perturbative spin-spin potential. It gives  $M(B^*) - M(B) = 45.7(3)$  MeV,  $M(B_s^*) - M(B_s) = 46.7(3)$  MeV ( $n_f = 4$ ), while in bottomonium  $\Delta_{HF}(b\bar{b}) = M(\Upsilon(9460)) - M(\eta_b(1S)) = 63.4$  MeV for  $n_f = 4$  and 71.1 MeV for  $n_f = 5$  are obtained; just latter agrees with recent BaBar data. For unobserved excited states we predict  $M(\Upsilon(2S)) - M(\eta_b(2S)) = 36(2)$  MeV,  $M(\Upsilon(3S)) - M(\eta(3S)) = 28(2)$  MeV, and also  $M(B_c^*) = 6334(4)$  MeV,  $M(B_c(2S)) = 6868(4)$  MeV,  $M(B_c^*(2S)) = 6905(4)$  MeV. The mass splittings between  $D(2^3S_1) - D(2^1S_0)$ ,  $D_s(2^3S_1) - D_s(2^1S_0)$  are predicted to be  $\sim 70$  MeV, which are significantly smaller than in several other studies.

## I. INTRODUCTION

Spin-spin interaction in mesons has been studied in a large number of theoretical papers [1]-[6], however, up to now some characteristic features of this interaction are not fully understood. This statement can be illustrated by theoretical failure to explain two experimental facts: rather small  $\psi(3686) - \eta_c(2S)$  mass difference:  $M(\psi(3686)) - M(\eta_c(2S)) = 49 \pm 4$  MeV [7]-[9] and, on the contrary, unexpected large HFS in bottomonium, which follows from the mass  $M(\eta_b(1S)) = 9391.1 \pm 3.1$  MeV of the  $\eta_b(1S)$  meson, recently discovered by the BaBar Collab. [10]. The  $\eta_b$  meson was observed in the radiative decays,  $\Upsilon(3S) \rightarrow \gamma \eta_b(1S)$  and  $\Upsilon(2S) \rightarrow \gamma \eta_b(1S)$  [10], and later confirmed by the CLEO Collaboration, also in the radiative  $\Upsilon(3S) \rightarrow \gamma \eta_b(1S)$  decay [11]. The measured value,  $\Delta_{HF}(b\bar{b}) = M(\Upsilon(1S)) - M(\eta_b(1S)) = 69.9 \pm 3.1$  MeV, is significantly larger than in most theoretical predictions, thus illustrating that modern understanding of HF interaction in QCD remains incomplete.

In perturbative approach a spin-spin potential between heavy quarks contains the factors, like the strong coupling and quark masses, which differ in different models and as a result, theoretical predictions for  $\Delta_{HF}(b\bar{b}) = M(\Upsilon(9460)) - M(\eta_b(1S))$  vary in wide range: 35 – 90 MeV [1]- [6], [12], being in most cases smaller than experimental number.

On fundamental level spin-spin potential  $V_{ss}$  has been recently studied in quenched QCD on large lattice [13], where this potential was shown to be compatible with zero at distances  $r \geq 0.30$  fm (for unknown reason at smaller  $r$  it has negative sign with a large magnitude). Although the lattice HF potential remains undefined at small  $r$ , its behavior at larger  $r$  is in agreement with widely used Fermi-Breit potential containing  $\delta^3(\vec{r})$  [14]. What is important that in lattice QCD, as well as in Field Correlator Method (FCM) [15]-[17], a spin-spin potential is described by universal functions, expressed via the field correlators. Moreover, in [17] it was shown that nonperturbative HF potential can give not small contributions to HFS.

On the other hand in Ref. [1] a smearing procedure for the  $\delta^3(\vec{r})$ -function was shown to be very important, giving a large Gaussian smearing parameter for heavy mesons, containing a  $b$  quark, so that for the  $B_q$  ( $q = n, s, c$ ) mesons and bottomonium a smearing occurs at very small distances and for them the use of the Fermi-Breit potential may be a good approximation. For the mesons, not containing a  $b$ -quark, a smearing parameter is essentially smaller, both for light mesons and for the  $D, D_s$  mesons [1]. However, such dependence of a smearing parameter on a quark content does not agree with the lattice and FCM representation about a spin-spin potential as a universal one in static approximation, where is defined by the field correlators with universal parameters [13], [17].

Recently HFS in the  $B_q$  mesons and quarkonia have also been calculated in lattice QCD [18]-[22] and their results we shall shortly discuss in our paper. Here we study HFS with the use of FCM, where both perturbative and nonperturbative spin-spin potentials are presented in analytical form [16], [17] and it allows to analyse the role of different physical parameters, defining HF structure. However, a comparison of our and lattice results is rather difficult, because in lattice calculations perturbative and nonperturbative spin-spin effects are not separated and a characteristic

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value of the strong coupling  $\alpha_{HF}$  is not discussed. On the contrary, our analysis shows that in bottomonium and the  $B_q$  ( $q = n, s, c$ ) mesons HFS can be described within only perturbative approach, since nonperturbative spin-spin potential gives a small contribution.

It is important that in the  $B_q(q = n, s)$  mesons and bottomonium a good agreement with experiment is reached taking a universal coupling,  $\alpha_{HF} = 0.310$  [23]. We would like to stress that this number is significantly larger than that prescribed in pQCD, where  $\alpha_s(m_b) \sim 0.18$  and  $\alpha_s(m_c) \sim 0.23$  are used; just because of such small coupling small HFS was predicted in bottomonium in [5], [12].

However, for charmonium and the  $D, D_s$  mesons their HFS turn out to be by  $\sim 15\%$  and  $\sim 30\%$  smaller than in experiment, if the same  $\alpha_{HF} = 0.310$  is taken. It can occur for two reasons: if for those states nonperturbative HF potential gives essential contributions, or higher order corrections are not small, as it takes place in fine structure splittings of the  $1P$  charmonium multiplet [24]. The situation is different for the charmonium excited states, for which just with a universal coupling,  $\alpha_{HF} \sim 0.31$ , a good agreement with experimental HFS:  $M(\psi(3686)) - M(\eta_c(3637)) = 49 \pm 4$  MeV [7] is obtained. Notice that the scale, corresponding to  $\alpha_{HF}(\mu) = 0.31 \simeq \alpha_s(\mu)$  is rather large,  $\mu \sim 1.7$  GeV.

In theoretical models two typical choices of  $\alpha_{HF}$  are used:

1. First one, when "a universal"  $\alpha_{HF}$  is used. For example, in [2]  $\alpha_{HF} = 0.36$  was taken from the fit to the mass difference,  $M(J/\psi) - M(\eta_c(1S)) = 117$  MeV; then for this choice predicted HFS in bottomonium,  $M(\Upsilon(9460)) - M(\eta_b(1S)) = 87$  MeV, has appeared to be by  $\sim 25\%$  larger than experimental number. In [3] a smaller universal  $\alpha_{HF} = 0.339$  was used in the heavy-light mesons; however, it is difficult to compare our and their results, because in [3] a large string tension,  $\sigma = 0.257 \text{ GeV}^2$ , was used, while here (as well as in [1]) the conventional  $\sigma = 0.18 \text{ GeV}^2$  is taken.
2. Second choice is mostly used in pQCD [5], [12], where a scale  $\mu = m_Q$  depends on a heavy quark mass and therefore the value of  $\alpha_{HF}(\mu) \simeq \alpha_s(m_Q)$  is essentially smaller. In [12], as well as in the EFG paper [4], just due to the choice of  $\alpha_{HF}(m_b) = 0.18$  small HFS were obtained in bottomonium (although the w.f. at the origin from [12] have provided a precision description of dielectron widths for  $\Upsilon(nS)$  ( $n = 1, 2, 3$ ) [25]).

In FCM a spin-spin potential takes into account relativistic corrections and the current masses are used for a light quark,  $m_n \sim 5$  MeV ( $n = u, d$ ), and  $m_s \simeq 200$  MeV for a  $s$  quark (about a choice of  $m_s$  see [26]), so that the  $B, D$ , and  $B_s, D_s$  mesons can be considered on the same footing as heavy quarkonia and the  $B_c$  mesons.

We shall show here that HFS are sensitive to the value of the vector QCD constant  $\Lambda_V(n_f)$ , defining a vector part of a static potential in coordinate space. In its turn this constant is expressed via  $\Lambda_{\overline{MS}}(n_f)$  [27], which at present are known with a good accuracy only for  $n_f = 5$  and with 10% accuracy for  $n_f = 3, 4$  [7]. To fix  $\Lambda_V(n_f)$  we assume here, as well as in [1], that in the one-gluon-exchange potential (OGE) the freezing value of the vector coupling  $\alpha_V(r)(n_f)$  is the same for  $n_f = 3, 4, 5$  (it is denoted as  $\alpha_{crit}$ ). Due to such an assumption the HFS dependence on  $n_f$  is weakening, with an exception of the bottomonium ground states.

We also calculate HFS and the masses of undiscovered yet mesons:  $\eta_b(2S)$ ,  $\eta_b(3S)$ ,  $B_c^*(1S)$ , and the masses of  $B_q(2S)$ ,  $D(2S)$ ,  $D_s(2S)$ .

The paper is organized as follows. In Section II the spin-spin potential is given in the form, where relativistic corrections are taken into account, as it is prescribed in FCM. Also relativistic string Hamiltonian is presented. In Section III the details of the static potential are discussed. In Section IV calculated w.f. at the origin and HFS for the  $B_q$  mesons and bottomonium are given and their dependence on the number of flavors is discussed. In Section V a choice of the strong coupling for the  $D, D_s$  mesons, and charmonium is discussed. Conclusions of our analysis are given in Section VI. In Appendix A the conventional formula for the pole mass of a heavy quark is shortly discussed and in Appendix B we describe the self-energy contribution to the meson mass, which is important for heavy-light mesons.

## II. THE HF POTENTIAL IN THE FIELD CORRELATOR METHOD

The conventional form of the Fermi-Breit potential [13],

$$\hat{V}_{ss}(r) = \mathbf{s}_1 \mathbf{s}_2 \frac{32\pi}{9} \frac{\alpha_{HF}(\mu)}{\tilde{m}_1 \tilde{m}_2} \delta^3(\mathbf{r}), \quad (1)$$

is widely used in heavy quarkonia, as well as in many nonrelativistic models. It contains the constituent quark masses  $\tilde{m}_1$  and  $\tilde{m}_2$ , which are model-dependent and can differ by  $\sim 30\%$ , or even larger, in different models, e.g. the mass of

a  $c$ -quark,  $m_c = 1.48$  GeV, was taken in [2], while in the Cornell potential a larger value,  $m_c = 1.84$  GeV, was used [28]. A constituent mass is supposed to be the same for all  $nS$  and  $nL$  states.

In Eq. (1) the strong coupling  $\alpha_{HF}(\mu)$  can differ from the QCD strong coupling  $\alpha_s(\mu)$  (in the  $\overline{MS}$  renormalization scheme) due to higher order perturbative corrections. These higher order corrections in one-loop approximation were calculated for heavy quarkonia [29]:

$$\alpha_{HF}(\mu) = \alpha_s(\mu) \left[ 1 + \frac{\alpha_s(mu)}{\pi} \rho(n_f) \right], \quad (2)$$

but remain unknown for heavy-light mesons, containing a light (or a strange) quark. Therefore in general case the coupling  $\alpha_{HF}$  in Eq.(1) should be considered as an effective one. Notice that its value is smaller than a freezing constant of the vector coupling  $\alpha_V(r)$ , which defines the OGE potential at large distances (or at small momenta) (see Eq.(17)). In heavy quarkonia with  $m_1 = m_2 = m_Q$  the factor  $\rho$  is known [29]:

$$\rho = \frac{5}{12}\beta_0 - \frac{8}{3} - \frac{3}{4}\ln 2 \quad (3)$$

and appears to be small:  $\sim 6 - 8\%$  for  $n_f = 3$ ,  $\sim 3 - 4\%$  for  $n_f = 4$ , and  $\leq 0.1\%$  for  $n_f = 5$ ; still in some cases these corrections can improve an accuracy of calculations. However, since they are not defined for heavy-light mesons, here in all cases we will consider  $\alpha_{HF}$  as an effective coupling, which is factually a fitting parameter.

The important role of relativistic corrections, even for the  $B_c$  meson, has been underlined in [1], [3], and also in the lattice calculations of  $B_c^*$  in full QCD [21]. In FCM relativistic corrections are taken into account in two ways: firstly, through the kinetic energies of a quark and antiquark, which enter a spin-spin potential [16], [17]:

$$\hat{V}_{ss}(r) = \mathbf{s}_1 \mathbf{s}_2 \frac{32\pi}{9} \frac{\alpha_{HF}(\mu)}{\omega_1 \omega_2} \delta(\mathbf{r}). \quad (4)$$

For this potential a HFS is

$$\Delta_{hf}(nS) = \frac{8}{9} \frac{\alpha_{HF}(\mu)}{\omega_1 \omega_2} |R_n(0)|^2, \quad (5)$$

where relativistic corrections are taken into account via the averaged kinetic energies  $\omega_1(nS), \omega_2(nS)$ :

$$\omega_1(nS) = \langle \sqrt{\mathbf{p}^2 + m_1^2} \rangle_{nS}, \quad \omega_2(nS) = \langle \sqrt{\mathbf{p}^2 + m_2^2} \rangle_{nS}, \quad (6)$$

which are well defined. By definition they depend on the quantum numbers of a given state  $nS$ , growing for larger  $nS$  states. The important point is that in (6) the masses  $m_1, m_2$  are not arbitrary (or fitting parameters): they are equal the pole masses  $m_c, m_b$  in heavy quarkonia, which are now known with an accuracy  $\sim 70$  MeV for a  $b$  quark and  $\sim 100$  MeV for a  $c$  quark (see [7] and references therein). In leading order the pole masses  $m_Q$  do not depend on a number of flavors, while in the order  $(\alpha_s(\bar{m}_Q)^2)$  they slightly depend on  $n_f$  (as in Eqs.(A.1) and (A.2) in Appendix A). For heavy quarks we take the following pole masses:  $m_c = 1.41$  GeV,  $m_b = 4.79$  GeV for  $n_f = 4$  and  $m_b = 4.82$  GeV for  $n_f = 5$ .

For a light quark ( $n = u, d$ ) we use the current mass  $m_n = 5$  MeV and  $m_s = 200$  MeV for a  $s$ -quark. The mass of a  $s$  quark is relatively large (close value of  $m_s$  is used in the Dirac equation in [3]), because the spectra of the  $D_s, B_s$  mesons are defined at the scale,  $\mu \leq 1$  GeV [26], which is smaller than the conventional scale 2 GeV, for which  $m_s(2 \text{ GeV}) \simeq 90$  MeV [7].

For excited states the kinetic energies  $\omega_i$  (6) increase and therefore HFS, calculated with the HF potential (4), are always smaller than those for the Fermi-Breit potential (1) with fixed (constituent) masses.

Other type of relativistic corrections enter via the w.f. at the origin, which together with the  $\omega_i$  are calculated from the relativistic string Hamiltonian (RSH)  $H_0$ , also derived in FCM [30],

$$H_0 = \frac{\omega_1}{2} + \frac{\omega_2}{2} + \frac{m_1^2}{2\omega_1} + \frac{m_2^2}{2\omega_2} + \frac{\mathbf{p}^2}{2\omega_{red}} + V_B(r). \quad (7)$$

The variables  $\omega_i$  enter  $H_0$  as the kinetic energy operators. However, while a HF interaction (as well as any spin-dependent potential) is considered as a perturbation, then in (4), (5)  $\omega_i$  should be changed by the matrix elements (m.e.) (6) [16].

From RSH a simple expression follows for a spin-averaged mass  $M(nS)$  [31]:

$$M(ns) = \frac{\omega_1}{2} + \frac{m_1^2}{2\omega_1} + \frac{\omega_2}{2} + \frac{m_2^2}{2\omega_2} + E_{nS}(\omega_{red}). \quad (8)$$

In (8) an excitation energy  $E_{nS}(\omega_{red})$  depends on the reduced mass:  $\omega_{red} = \frac{\omega_1\omega_2}{\omega_1+\omega_2}$ . Also in bottomonium the mass formula (8) does not contain any overall constant, while for heavy-light mesons a negative (with not small magnitude) self-energy term, proportional to  $(\omega_q)^{-1}$  ( $q = n, s$ ), should be added [32] (see the expression (A.3) in Appendix B).

We use here the Einbein Approximation (EA), when the variables  $\omega_i(nS)$ , the excitation energy  $E_{nS}(\omega_{red})$ , and the w.f. are calculated from the Hamiltonian (6) and two extremum conditions,  $\frac{\partial M(nS)}{\partial \omega_i} = 0$  ( $i = 1, 2$ ), which are put on the mass  $M(nS)$  [31]:

$$\left[ \frac{\omega_1}{2} + \frac{\omega_2}{2} + \frac{m_1^2}{2\omega_1} + \frac{m_2^2}{2\omega_2} + \frac{\mathbf{p}^2}{2\omega_{red}} + V_B(r) \right] \varphi_{nS}(r) = E(nS) \varphi_{nL}, \quad (9)$$

$$\omega_i^2(nS) = m_i^2 - 2\omega_i^2 \frac{\partial E(nS, \mu_{red})}{\partial \omega_i(nS)} \quad (i = 1, 2). \quad (10)$$

Before to define the w.f. at the origin and  $\omega_i(nS)$  in next Section we shortly discuss the static potential  $V_B(r)$  in the Eq.(9).

### III. THE STATIC POTENTIAL $V_B(r)$

In a Hamiltonian approach a choice of a static potential  $V_B(r)$  is of a special importance; we take it as a sum of linear confining term and the OGE -type term: this additivity of a static potential is well established now in analytical studies [33] and on lattice [34], [35]:

$$V_B(r) = \sigma r + \frac{4\alpha_B(r)}{3r}. \quad (11)$$

For the string tension we use the conventional value,  $\sigma = 0.18 \text{ GeV}^2$ , for all mesons (if their sizes are less than  $\sim 1$  fm. This our choice is in contrast to that in [3], where in the Dirac equation large  $\sigma = 0.257 \text{ GeV}^2$  was used for heavy-light mesons.

The OGE term contains the vector coupling  $\alpha_V(r)$ , taken here in a particular case from the background perturbation theory (BPT) and denoted as  $\alpha_B(r)$  [36], [37]. Two important conditions have to be put on a vector coupling:

1. i) As in pQCD, it has to possess the asymptotic freedom (AF) property; just due to this property a static interaction depends on a number of flavors. Also the AF behavior strongly affects the w.f. at the origin.
2. ii) The vector coupling freezes at large distances. The property of freezing was widely used in phenomenology [1]-[3] and confirmed in lattice calculations of a static potential [34], where a freezing property was assumed at rather small quark-antiquark separations,  $r \geq 0.2 \text{ fm}$ .

On phenomenological level the freezing phenomenon has been suggested in [38], where in momentum space the logarithm  $\ln \frac{q^2}{\Lambda^2}$  in  $\alpha_s(q^2)$  was changed by  $\ln \frac{q^2 + 4m_g^2}{\Lambda^2}$ , thus introducing a regulator  $4m_g^2$ . The mass  $m_g$  was interpreted as an effective gluon mass, although a meaning of  $m_g$  is not well defined, since in QCD a gluon has no a mass. Later a freezing phenomenon was studied in BPT [37], where this regulator was shown to be equal a mass of the lowest hybrid excitation (called the background mass), with  $M_B = 1.0 \pm 0.05 \text{ GeV}$  [39] for  $n_f = 4, 5$  and a larger value,  $M_B \sim 1.5 \text{ GeV}$  for  $n_f = 0$  [36]. As in [1], we shall call a freezing constant a critical one and denote it as  $\alpha_{crit}$ .

Unfortunately, the critical constants, calculated from the static potentials on lattice, are significantly smaller than those in phenomenology and BPT. Here in BPT we use rather large  $\alpha_B(crit) = 0.58 - 0.60$  for  $n_f = 4, 5$ , which are close to  $\alpha_{crit} = 0.60$  (for any  $n_f$ ) taken in [1]. On lattice the freezing effect occurs at small distances,  $r \geq 0.2 \text{ fm}$ , and small  $\alpha_{crit}(lat) \simeq 0.30$  in full QCD ( $n_f = 3$ ) [35] and  $\alpha_{crit}(lat) \simeq 0.22$  in quenched calculations ( $n_f = 0$ ) [34] were obtained. The reasons of these discrepancies are not established yet.

While the critical value  $\alpha_{crit}$  is fixed, then with the use of Eq.(17) the constant  $\Lambda_B(n_f)$  (for a given  $n_f$ ) can be defined. It is important that this constant cannot be considered as a fitting parameter, because it is expressed via the QCD constant  $\Lambda_{\overline{MS}}(n_f)$  in the  $\overline{MS}$  renormalization scheme [27] (see the relation (16)). Therefore one can state that  $\Lambda_{\overline{MS}}$  indirectly defines  $\alpha_{crit}$ .

In the OGE term (11) a vector coupling in coordinate space  $\alpha_B(r)$  is defined through the vector coupling  $\alpha_B(q^2)$  in the momentum space [25], [36]:

$$\alpha_B(r) = \frac{2}{\pi} \int_0^\infty dq \frac{\sin(qr)}{q} \alpha_B(q). \quad (12)$$

Here the vector coupling  $\alpha_B(q^2)$  is taken in two-loop approximation,

$$\alpha_B(q) = \frac{4\pi}{\beta_0 t_B} \left( 1 - \frac{\beta_1}{\beta_0^2} \frac{\ln t_B}{t_B} \right), \quad (13)$$

where the logarithm,

$$t_B = \frac{q^2 + M_B^2}{\Lambda_B^2}, \quad (14)$$

contains the constant  $\Lambda_B(n_f)$  defined via the QCD constant  $\Lambda_{\overline{MS}}(n_f)$ . The relation between them has been established in [27]:

$$\Lambda_B(n_f) = \Lambda_{\overline{MS}} \exp \left( -\frac{a_1}{2\beta_0} \right), \quad (15)$$

with  $\beta_0 = 11 - \frac{2}{3}n_f$  and  $a_1 = \frac{31}{3} - \frac{10}{9}n_f$ . From the relation (15) one can see that for a given  $n_f$  the constant  $\Lambda_B(n_f)$  is significantly larger than  $\Lambda_{\overline{MS}}$ :

$$\begin{aligned} \Lambda_B^{(5)} &= 1.3656 \Lambda_{\overline{MS}}^{(5)} \quad (n_f = 5); \\ \Lambda_B^{(4)} &= 1.4238 \Lambda_{\overline{MS}}^{(4)} \quad (n_f = 4); \\ \Lambda_B^{(3)} &= 1.4753 \Lambda_{\overline{MS}}^{(3)} \quad (n_f = 3). \end{aligned} \quad (16)$$

At present the QCD constant  $\Lambda_{\overline{MS}}^{(5)}$  (for  $n_f = 5$ ) is known from experimental value of  $\alpha_s(M_Z) = 0.1182 \pm 0.0012$  [7]. Then in two-loop approximation it gives  $\Lambda_{\overline{MS}}(\text{two-loop}) = 232(12)$  MeV. The QCD constants  $\Lambda_{\overline{MS}}$  for  $n = 3, 4$  are extracted from experiments with lower accuracy,  $\sim 10\%$  [7]. To define them we fix here the freezing value  $\alpha_{crit}$ , assuming that they are the same for  $n_f = 3, 4, 5$ . Then from  $\alpha_{crit}$  (17) one can calculate all constants  $\Lambda_B(n_f)$  and then from (16) to define  $\Lambda_{\overline{MS}}(n_f)$ .

Notice that the mass  $M_B$  depends on  $\sigma$ , being proportional to  $\sqrt{\sigma}$ , and for  $\sigma = 0.18 \text{ GeV}^2$  the value,  $M_B = 1.0 \pm 0.05 \text{ GeV}$ , was extracted from a detailed comparison of the static force in FCM and lattice QCD [39] and also the analysis of the bottomonium spectra [40]. Here we use  $M_B = 0.95 \text{ GeV}$ . We also use here two values of  $\Lambda_{\overline{MS}}^{(5)}$ , equal 236 MeV and 245 MeV, which give the critical couplings 0.58 and 0.605.

From (12) it can be easily shown that the critical couplings in momentum and coordinate space coincide,  $\alpha_B(crit) = \alpha_B(q^2 = 0) = \alpha_B(r \rightarrow \infty)$ , and it is given by the expression [36]:

$$\alpha_B(crit) = \alpha_B(r \rightarrow \infty) = \alpha_B(q = 0) = \frac{4\pi}{\beta_0 t_0} \left( 1 - \frac{\beta_1}{\beta_0^2} \frac{\ln t_0}{t_0} \right), \quad (17)$$

with  $t_0 = t_B(q^2 = 0) = \ln \left( \frac{M_B^2}{\Lambda_B^2} \right)$ .

From (16) and  $\Lambda_{\overline{MS}}^{(5)} = 245(236) \text{ MeV}$  one obtains that in two-loop approximation  $\alpha_s(M_Z) = 0.1194(0.1188)$ , which agrees within an error with the world average  $\alpha_s(M_Z) = 0.1184 \pm 0.0012$  [7]. In Table I we summarize the values of  $\Lambda_B, \Lambda_{\overline{MS}}$  for  $n_f = 3, 4, 5$ .

The solutions of the coupled equations (9), (10), like the spectra and w.f at the origin, have been checked in numerous studies [23]-[26], [39]-[40], demonstrating a good description of different physical characteristics.

#### IV. RELATIVISTIC CORRECTIONS AND THE WAVE FUNCTIONS AT THE ORIGIN

With the use of the Eqs. (9) and (10) the kinetic energies  $\omega_i$  are calculated (see Table II). Their numbers show that relativistic corrections are small in bottomonium:  $\omega_b(1S) - m_b \simeq 180 \text{ MeV}$  ( $\sim 4\%$ ) and a bit larger,  $\sim 7\%$ ,

TABLE I: The vector constants  $\Lambda_B$  and  $\Lambda_{\overline{MS}}$  (in MeV) ( $n_f = 3, 4, 5$ ) for  $\alpha_{crit} = 0.605$ .

$n_f$	3	4	5
$\Lambda_B$	400	372	335
$\Lambda_{\overline{MS}}$	271	261	245

TABLE II: The kinetic energies  $\omega_q(1S)$  ( $q = n, s, c$ ) and  $\omega_b(1S)$  (in MeV) for the static potential  $V_B(r)$  (11) with  $n_f = 4$ ,  $\Lambda_B = 372$  MeV,  $\alpha_{crit} = 0.605$ .

Meson	$B$	$B_s$	$B_c$
$m_q$	5	200	1410
$\omega_q - m_q$	633	486	340
$m_b$	4790	4790	4820
$\omega_b - m_b$	42	46	110

for the  $2S, 3S$  states. It is of interest to notice that a relativistic correction to the  $b$ -quark mass in the  $B_c$  meson is even smaller than in bottomonium,  $\omega_b(1S) - m_b \simeq 110$  MeV ( $\sim 2\%$ ), while for a  $c$ -quark in  $B_c$  the difference,  $\omega_c(1S) - m_c \simeq 340$  MeV, is already  $\sim 25\%$  (all numbers refer to  $\alpha_{crit} = 0.605$ ,  $n_f = 4$ ).

For the  $B_q$  mesons ( $q = n, s, c$ ) relativistic corrections,  $\omega_q(1S) - m_q$  are given in Table II.

In charmonium relativistic corrections to a  $c$ -quark mass are already  $\sim 13\%$  for the ground state and  $\sim 17\%$  for the  $2S$  state. It is of interest to notice that they are even smaller,  $\sim 7\%$  for a  $c$  quark in the  $D, D_s$  mesons (see Table III).

For the analysis of HFS it is convenient to introduce the ratio  $g_{B_q}, g_{D_q}$ , and also  $g_b \equiv g(b\bar{b}), g_c \equiv g(c\bar{c})$ ,

$$g_{B_q}(nS) = \frac{|R_n(0)|^2}{\omega_1(nS)\omega_2(nS)}, \quad (18)$$

which directly enters the HFS (4):

$$\Delta_{HF}(nS) = \frac{8}{9} \alpha_{HF}(\mu) g_{B_q}(nS) \quad (19)$$

and appears to be weakly dependent on small variations of the masses  $m_1, m_2$ , and other parameters of a static potential, which are compatible with a good description of the meson spectrum. Their values for the  $B_q$  mesons and bottomonium are given in Tables IV and V.

In general case the w.f. at the origin is sensitive to the choice of  $\Lambda_B(n_f)$ , defining the OGE term, and may be  $\sim 1.5$  times larger, if the AF behavior is neglected [28]. In our calculations, when the AF property is taken into account and the same  $\alpha_{crit}(n_f)$  is used for  $n_f = 3, 4, 5$ , the differences in the  $g_{B_q}, g_{D_q}$  values turn out to be small,  $\leq 5\%$  for different  $n_f$ , (see Table IV), with exception of  $g_b$  in bottomonium.

Below we give the values of  $g_b(n_f)$  for  $n_f = 3, 4, 5$ , which were calculated with the same  $\alpha_{crit} = 0.605$ . For this  $\alpha_{crit}$  and  $M_B = 0.95$  GeV in two-loop approximation the constants  $\Lambda_B(n_f = 3) = 0.40$  GeV,  $\Lambda_B(n_f = 4) = 0.372$  GeV, and  $\Lambda_B(n_f = 5) = 0.335$  GeV are easily calculated (see Table I). The largest difference takes place in bottomonium, where

$$g_b(n_f = 3) = 0.213 \text{ GeV}, \quad g_b(n_f = 4) = 0.230 \text{ GeV}, \quad g_b(n_f = 5) = 0.258 \text{ GeV}, \quad (20)$$

TABLE III: The kinetic energies  $\omega_q(nS)$  ( $q = n, s$ ) and  $\omega_c$  (in MeV) for the static potential  $V_B(r)$  ( $n_f = 4$ ) with the same parameters as in Table II.

Meson	$D$	$D_s$	$c\bar{c}(1S)$	$c\bar{c}(2S)$
$m_q$	5	200	1410	1410
$\omega_q - m_q$	542	400	184	245
$\omega_c - m_c$	102	108	184	245

TABLE IV: The ratios  $g_{B_q}(1S)$  (in GeV) and  $|R_1(0)|^2$  (in  $\text{GeV}^3$ ) for the  $B_q(1S)$  mesons ( $\alpha_{crit} = 0.605$ ,  $\Lambda_B(n_f = 4) = 372$  MeV,  $\Lambda_B(n_f = 3) = 400$  MeV).

	$B$	$B_s$	$B_c$
$ R(0) ^2(n_f = 4)$	0.510	0.558	1.742
$g_{B_q}(n_f = 4)$	0.165	0.168	0.212
$ R(0) ^2(n_f = 3)$	0.478	0.527	1.626
$g_{B_q}(n_f = 3)$	0.158	0.161	0.198

TABLE V: The ratios  $g_b(nS)$  (in GeV),  $|R_n(0)|^2$  (in  $\text{GeV}^3$ ), and HFS  $\Delta_{HF}(nS)$  (in MeV) for the  $1S, 2S, 3S$  bottomonium states ( $n_f = 4, 5$ ,  $\alpha_{crit} = 0.605$ ).

	1S	2S	3S
$ R_n(0) ^2(n_f = 5)$	6.476	3.398	2.682
$g_b(n_f = 5)$	0.258	0.134	0.105
$\Delta_{HF}(n_f = 5)$	71.1	36.9	28.9
$ R_n(0) ^2(n_f = 4)$	5.668	3.126	2.508
$g_b(n_f = 4)$	0.230	0.127	0.100
$\Delta_{HF}(n_f = 4)$	63.4	35.0	27.6

i.e.  $g_b(n_f)$  can change by  $\sim 20\%$ . Therefore, if experimental HFS in bottomonium are known with a precision accuracy, then one can distinguish between cases with different  $n_f$ . From the numbers given in (20) and taking  $\alpha_{HF} = 0.310$ , as for the  $B_q$  mesons, one obtains the following HFS  $\Delta_{HF}(b\bar{b}) = M(\Upsilon(9460)) - M(\eta_b(1S))$ :

$$\Delta_{HF}(n_f = 3) = 58.7 \text{ MeV}; \quad \Delta_{HF}(n_f = 4) = 63.4 \text{ MeV}; \quad \Delta_{HF}(n_f = 5) = 71.1 \text{ MeV} \quad (21)$$

Notice that our value for the  $\Upsilon(9460) - \eta_b(1S)$  mass difference with  $n_f = 3$  has appeared to be in good agreement with the lattice calculations, also performed with  $n_f = 3$ , and where  $\Delta_{HF}(b\bar{b}) = 61 \pm 13 \pm 4$  MeV was obtained in Ref.[18];  $70 \pm 11$  MeV in [19], and a smaller splitting,  $\Delta_{HF}(b\bar{b}) = 54 \pm 12$  MeV, was calculated in [20]. (An accuracy of our calculations for HFS is estimated to be  $\pm 4$  MeV ( $\sim 5\%$ ), as it follows by varying different parameters, i.e. is better than in lattice calculations, where at present it is  $\geq 20\%$ ).

For the  $B_q$  mesons their w.f. at the origin, the factors  $g_{B_q}$ , and the HFS are given in Table IV for  $n_f = 3, 4$ . As seen from Table IV, for the  $B_q$  mesons a difference between  $g_{B_q}(n_f = 3)$  and  $g_{B_q}(n_f = 4)$  appear to be only by  $\sim 4\%$ ; also for a given  $n_f$  a difference between  $g_B$  and  $g_{B_s}$  is small,  $\leq 2\%$ .

In bottomonium a difference between  $g_b(n_f = 3)$  and  $g_b(n_f = 4)$  is larger,  $\sim 12\%$ , and in both cases they are smaller than  $g_b(n_f = 5)$ ; corresponding HFS are given in Table V. For excited states the  $\Upsilon(nS) - \eta_b(nS)$  splittings ( $n = 2, 3$ ) are given also for the coupling  $\alpha_{HF} = 0.310$  and this choice seems to be a realistic, because in bottomonium a characteristic momentum weakly changes for excited states:

$$\omega_b(1S) - m_b = 185(5) \text{ MeV}, \quad \omega(2S) - m_b = 195(5) \text{ MeV}, \quad \omega_b(3S) - m_b = 225(5) \text{ MeV}. \quad (22)$$

In (22) theoretical errors given come from slightly different pole mass of a  $b$  quark for different  $n_f$ .

Finally in Tables VI, VII we give  $g_D, g_{D_s}$  for ground states, and also  $g_c(nS) \equiv g_{c\bar{c}}(nS)$  for the  $1S$  and  $2S$  charmonium states. It is worthwhile to notice that the factor  $g_{c\bar{c}}(2S)$  is two times smaller than  $g_{c\bar{c}}(1S)$ , being one of the reasons why the mass difference  $\psi(3686) - \eta_c(2S)$  is small.

We would like to remind here that while the relativistic string Hamiltonian (7) is used, for a meson excited states may be considered on the same footings as a ground state till, if a single-channel approximation can be applied. Otherwise one has to use the multichannel Hamiltonian, also derived within FCM [41].

TABLE VI: The w.f.  $|R(0)|^2$  (in  $\text{GeV}^3$ ) and the ratios  $g_D, g_{D_s}, g_c(1S), g_c(2S)$  (in GeV) for the  $D, D_s$  mesons and charmonium (in the static potential  $V_B(r)$   $\sigma = 0.18$   $\text{GeV}^2$ ,  $\Lambda_B(n_f = 3) = 0.40$  GeV,  $\Lambda_B(n_f = 4) = 0.372$  GeV,  $M_B = 0.95$  GeV).

	$D(1S)$	$D_s(1S)$	$c\bar{c}(1S)$	$c\bar{c}(2S)^a$
$ R(O) ^2$	0.314	0.346	0.856	0.476
$g(nS)$	0.379	0.379	0.340	0.174

<sup>a</sup>For the  $2S$  charmonium state in the w.f. at the origin the  $S - D$  mixing between  $\psi(3686)$  and  $\psi(3770)$  states is taken into account

TABLE VII: The HFS (in MeV) in the  $B_q$  mesons with  $\alpha_{HF}(n_f = 4) = 0.310$ ,  $\Lambda_B(n_f = 4) = 0.372$  GeV and  $\alpha_{HF}(n_f = 3) = 0.324$ ,  $\Lambda_B(n_f = 3) = 0.40$  GeV.

HFS	$B$	$B_s$	$B_c(1S)$
$\Delta_{HF}(n_f = 4)$	45.6	46.3	58.4
$\Delta_{HF}(n_f = 3)$	45.4	46.1	57.2
$\Delta_{HF}(\text{exp})$	$45.78 \pm 0.35$	$46.5 \pm 1.25$	abs

TABLE VIII: The masses  $M(B_q(2S))$ ,  $M(B_q^*(2S))$ , and the HFS  $M(B_q^*(2S) - M(B_q(2S)))$ , ( $q = n, s, c$ ) (in MeV) for  $\alpha_{HF}(n_f = 4) = 0.310$  and  $\alpha_{crit}(n_f = 4) = 0.605$ .

Meson	$B(2S)$	$B_s(2S)$	$B_c(2S)$
$g_{B_q}(2S)$	0.124	0.1255	0.1353
$\Delta_{hf}(B_q(2S))$	34.3	34.7	37.3
$M(B_q(2^1S_0))$	5967	6040	6868
$M(B_q^*(2^3S_1))$	6001	6075	6905

## V. HFS IN BOTTOMONIUM AND THE $B_q$ MESONS

Firstly, for bottomonium we compare calculated here HFS, Eq. (21), with experimental HFS, which follows from experimental mass of  $\eta_b(1S)$ :

$$\begin{aligned}\Delta_{HF}(b\bar{b}) &= M(\Upsilon(9460)) - M(\eta_b) = 69.9 \pm 3.2 \text{ MeV} \quad [10]; \\ \Delta_{HF}(b\bar{b}) &= 68.5 \pm 6.6 \text{ MeV} \quad [11].\end{aligned}\tag{23}$$

In the BaBar experiment the  $\eta_b$  mass is defined with a small errors,  $\pm 3.2$  MeV, and even with a smaller error,  $\leq 1$  MeV, the mass differences:  $M(B^*) - M(B)$ ,  $M(B_s^*) - M(B_s)$  are known now from experiment [7]. Such good experimental data allow to extract a coupling  $\alpha_{HF}$  with a good accuracy. For fitting procedure the important point is that in bottomonium a fitted value of  $\alpha_{HF}$  cannot be larger than that for  $B_q$  mesons, since due to the AF property a coupling of a smaller size system (bottomonium) is typically smaller. For the  $B_q$  mesons and bottomonium the best fit is obtained for  $\alpha_{HF} = 0.310$ , later called "a universal" coupling.

As seen from (21), in bottomonium full agreement with experiment is reached if  $\alpha_{HF} = 0.310$  and  $n_f = 5$  are taken in the static potential. It gives  $\Delta_{HF}(b\bar{b}) = 71.1$  MeV, which is by 8 MeV larger than  $\Delta_{HF}(b\bar{b}) = 63.4$  MeV for  $n_f = 4$ . (We do not consider as unphysical a fit with  $n_f = 4$  and  $\alpha_{HF} = 0.348$  when agreement with experiment is also possible, since the value of this coupling is by 12% larger than that for the  $B_q$  mesons). Notice that the best description of the bottomonium spectra takes place also for  $n_f = 5$  [40].

In Table V the HFS for the bottomonium  $2S, 3S$  states are given; they are equal 36(1) MeV and 28(1) MeV, respectively, being weakly dependent on  $n_f$  taken.

The HFS of the  $B_q$  ground states are presented in Table VII, while in Table VIII the mass splittings for higher states,  $B(2S)$ ,  $B_s(2S)$ ,  $B_c(2S)$ , are also given.

For the  $B^*(1S) - B(1S)$ ,  $B_s^*(1S) - B_s(1S)$  splittings a good agreement with experiment is reached in two cases: with  $n_f = 4$  and  $\alpha_{HF} = 0.310$  (as in bottomonium) and with  $n_f = 3$  and a bit larger  $\alpha_{HF} = 0.324$  (see Table VII). First choice seems to be preferable as a universal one, but in any case a difference between two couplings is small,  $\leq 5\%$ . Therefore one can speak about a universal  $\alpha_{HF}$  within 5% accuracy. To fix a preferable number  $n_f$  for the  $B, B_s$  mesons one needs to use an additional information, like the decay constants etc.

For the  $B_c$  mesons our splitting,  $M(B_c^*) - M(B_c) = 58.4$  MeV for  $n_f = 4$  and 57.2 MeV for  $n_f = 3$ , appears to be in agreement with the unquenched lattice calculations from [21], where the number  $53 \pm 7$  MeV is predicted.

For excited  $B_c(2S)$  states our calculations give the centroid mass,  $M_{cog}(B_c(2S)) = 6893$  MeV, and the  $B_c^*(2S) - B_c(2S)$  splitting 37.3 MeV (see Table VIII), from which  $M(B_c^*(2^3S_1)) = 6.902$  MeV and  $M(B_c(2^1S_0)) = 6865$  MeV. An accuracy of our calculations, performed in single-channel approximation, is estimated to be  $\pm 5$  MeV, although for higher states an influence of open channel(s) may be important.

From Table VIII one can see that for excited  $B, B_s, B_c$  mesons their HFS have close values,  $\Delta_{HF}(B_q(2S)) \sim 34 - 37$  MeV. and the masses of singlet and triplet  $2S$  states are also given in Table VIII. We would like to notice that



our HFS for the  $B_s$  mesons differ from those calculated in lattice QCD [22], where small  $\Delta_{HF}(B_s(1S)) = 29.8 \pm 3.2$  MeV was calculated for the ground  $1S$  states (in our calculations it is equal 46.3 MeV), while on the contrary, in [22] for the excited  $2S$  states a central value of the HFS,  $\Delta_{HF}(B_s(2S)) = 56 \pm 27$  MeV, is larger than in our calculations, where this splitting is only 35 MeV.

Our calculations have been performed in single-channel approximation with a string tension  $\sigma = \text{const} = 0.18$  GeV<sup>2</sup>. For higher levels a influence of open channels can be taken into account, e.g. via a flattening of the static potential, and then the masses  $M(B(2S)), M(B_s(2S))$  appear to be only by  $\sim 10$  MeV smaller. The effect from open channels may be more important for the  $D(2S), D_s(2S)$  states and then a multichannel relativistic Hamiltonian from [41] may be used instead of the equation (7).

Thus our analysis of HFS for the  $B(1S), B_s(1S)$  mesons and bottomonium shows that a good agreement with experiment is reached with a universal  $\alpha_{HF} = 0.310$ . This coupling is smaller than that in [2], [3] and corresponds to rather large renormalization scale,  $\mu \simeq 1.70$  GeV. This value of the scale confirms existing interpretation of the spin-spin potential as dominantly a short-range perturbative one, thus justifying the use of the  $\delta(\mathbf{r})$ -function.

## VI. LARGE HFS IN CHARMONIUM AND THE $D, D_s$ MESONS

Experimental HFS for the  $D, D_s$  ground states are large,  $\simeq 140$  MeV, being three times larger than those for the  $B, B_s$  mesons. Let us firstly estimate these HFS, taking the factors  $g_D = g_{D_s} = 0.379$  GeV from Table VI ( $n_f = 4$ ), and the value  $\alpha_{HF} = 0.310$ , as for the  $B_q$  mesons. Then for the ground states we obtain  $\Delta_{HF}(D(1S)) = \Delta_{HF}(D_s(1S)) = 104.4$  MeV, which are by 35% smaller than experimental HFS [7].

In charmonium a discrepancy between calculated HFS with  $\alpha_{HF} = 0.310$ ,  $\Delta_{HF}(1S, c\bar{c}) = 93.7$  MeV, and experimental number is smaller,  $\sim 20\%$ . If in charmonium first order correction (3), equal  $\leq 6\%$ , is taken into account, then this discrepancy remains not small,  $\sim 15\%$ .

For the  $2S$  charmonium states, a situation is different and full agreement with experimental HFS takes place, if a universal coupling  $\alpha_{HF} = 0.31$  is used. With the factor  $g_c(2S) = 0.174$  GeV from Table VI, one obtains

$$\Delta_{HF}(2S, c\bar{c}) = M(3686) - M(\eta_c(2S)) = 47.9, \text{ MeV} \quad (24)$$

coinciding with the experimental  $\psi(3686) - \eta_c(2S)$  mass difference:  $\Delta_{HF}(2S, c\bar{c})|_{exp} = 48 \pm 4$  MeV [7]. (To get this result we have taken into account the  $S - D$  mixing between  $\psi(3686)$  and  $\psi(3770)$  with the mixing angle  $\theta \sim 11^\circ$  [42]). Thus for the  $2S$  charmonium states a universal coupling provides agreement with experiment. Therefore we expect that for the  $2S$  states of other heavy-light mesons the coupling  $\alpha_{HF} = 0.31$  can be also used.

However, it remains unclear what kind of corrections ( $\sim 15\%$  in charmonium ground states) have been lost in our analysis? (We remind that first order perturbative contribution gives only  $\leq 6\%$ .) We assume here that such a contribution comes from nonperturbative spin-spin potential, and just due to nonperturbative correlators lattice calculations [18]-[20] give the mass difference,  $M(J/\psi) - M(\eta_c(1S))$ , in good agreement with experimental number, equal 117 MeV. Such HFS, coming from nonperturbative spin-spin potential, can be taken into account also within FCM, where recently new results have been obtained for the vacuum correlation functions and correlation lengths [17], [43]. Detailed analysis of these effects will be considered in our next paper.

However, we can use results from Ref.[25], that for higher states nonperturbative contributions are much smaller than for the ground states; just for that reason we have obtained a good description of HFS for the charmonium  $2S$  states with  $\alpha_{HF} = 0.31$ . Therefore we use here only perturbative part of HFS for higher states.

It is of interest to notice that for  $D(2S)$  and  $D_s(2S)$  their w.f. at the origin and the quark kinetic energies can differ by  $\sim 10\%$ , nevertheless the factors  $g_D(2S)$  and  $g_{D_s}(2S)$  coincide within 4% accuracy, being equal  $0.273 \pm 0.005$  GeV, if linear confining term is taken in the static potential; it gives  $\sim 75$  MeV for their HFS. If one takes into account flattening of confining potential, which is often important for higher states [44], then a small decreasing of this factor takes place:  $g_D(2S) \simeq g_{D_s}(2S) = 0.264 \pm 0.004$  GeV, where the error occurs due to possible different choice of  $n_f = 3$  or  $n_f = 4$ ; in this case the HFS are equal  $72 \pm 3$  MeV for the  $D(2S), D_s(2S)$  states. Calculated HFS are presented in Table IX.

As seen from Table IX, the HFS of the  $D(2S), D_s(2S)$  mesons,  $\sim 70$  MeV, are two times smaller than those for the ground states, i.e. for these states a picture is similar to that in charmonium, when the HFS for the  $2S$  states is 2.3 times smaller than the  $J/\psi - \eta_c(1S)$  mass difference.

For the  $D(2S), D_s(2S)$  mesons our HFS appear to be significantly smaller than the  $D_s(2^3S_1) - D_s(2^1S_0)$  mass difference, predicted in [45], where it is equal 151 MeV, and even larger  $\Delta_{HF}(D(2S)) = 188$  MeV and  $\Delta_{HF}(D_s(2S)) = 192$  MeV were obtained in [46]. On the contrary, in our approach the HFS for the  $2S$  states have appeared to be rather close to those from the GI paper [1], although in a static potential our and their sets of parameters are very much different, with an exception of the value of the string tension, equal 0.18 GeV<sup>2</sup> in both

TABLE IX: The HFS (in MeV) of the  $D(2S)$ ,  $D_s(2S)$  mesons, and charmonium with  $\alpha_{HF} = 0.31$ ; experimental HFS from [7].

Meson	$\Delta_{HF}$	$\Delta_{HF}(exp)$
$D(2S)$	$72 \pm 3$	abs
$D_s(2S)$	$72 \pm 3$	abs
$c\bar{c}(1S)$	93.7	$116.6 \pm 1.2$
$c\bar{c}(2S)$	47.9	$49 \pm 4$

TABLE X: The masses of the singlet and triplet  $2S$  states (in GeV) for  $D(2S)$  and  $D_s(2S)$  ( $\alpha_{HF} = 0.31$ ).

State	this paper	GI [1]	RRS [45]	MMS [46]
$D(2^1S_0)$	2.570	2.58	abs	2.483
$D(2^3S_1)$	2.642	2.64	abs	2.671
$D_s(2^1S_0)$	2.664	2.67	2.486	2.563
$D_s(2^3S_1)$	2.736	2.73	2.637	2.755

calculations. Comparison our results and predictions from Refs. [1], [45], [46], where different relativistic models used, are presented in Table X.

The masses, presented in Table X, were calculated with the use of different relativistic models, in particular the Spinless Salpeter Equation was exploited in [1], [45], and also in our calculations here, with the static potential described in Section 3. From Table X one can see that the singlet and triplet masses for the  $D(2S)$  and  $D_s(2S)$  mesons appear to be very close to each other in our calculations and in [1]. On the contrary, in [45] and [46] predicted HFS for the  $2S$  states are 2.0 and 2.6 times larger than in our calculations, and due to this result, their masses of a singlet state,  $M(D_s(2^1S_0))$ , is by  $\sim 100$  MeV and  $\sim 170$  MeV, respectively, lower than in [1] and in our analysis.

For the triplet  $2S$  states differences in predicted masses are not large and in [1], [46], and our calculations  $M(D^*(2S))$  and  $M(D_s^*(2S))$  lie in the range 2.64-2.67 GeV and 2.73-2.75 GeV, respectively. These predictions are in good agreement with the experimental mass,  $M_{exp}(D_s^*(2S)) = 2710 \pm 2 \pm_{-7}^{+12}$  MeV [47], [48], while in [45] predicted mass is by  $\sim 70$  MeV smaller.

From our analysis it follows that observation of the singlet states,  $D(2^1S_0)$  and  $D_s(2^1S_0)$ , is crucially important for understanding of spin-spin interaction in systems of large sizes, in particular, it could clarify what is a characteristic value of the strong coupling in HF interaction.

## VII. CONCLUSIONS

In our study we have used a conception of a universal HF interaction and observed that

1. In the  $B_q$  mesons and bottomonium a good agreement with experimental HFS are reached if a universal coupling  $\alpha_{HF} = 0.310$  is used in the HF potential.
2. Just with the same coupling,  $\alpha_{HF} = 0.310$ , the  $\psi(3686) - \eta_c(2S)$  mass splitting appears to be in agreement with experiment.
3. Calculated here mass splitting,  $M(B_c^*) - M(B_c) = 57.9(6)$  MeV, gives the mass of unobserved yet  $B_c^*$  meson,  $M(B_c^*) = 6.334 \pm 5$  MeV. Our HFS is close to that in full QCD calculations,  $\Delta_{HF}(B_c) = 53(7)$  MeV [21].
4. In bottomonium a full agreement with experimental mass of  $\eta_b(1S)$  is reached only if in the static potential  $n_f = 5$  is used, giving  $\Delta_{HF}(b\bar{b}) = 71.1$  MeV. For  $n_f = 4$  and  $n_f = 3$  calculated HFS are smaller, being equal 63.4 MeV and 58.7 MeV, in agreement with the lattice results for  $n_f = 3$ .
5. With  $\alpha_{HF} = 0.310$  for the bottomonium  $2S$  and  $3S$  states the HFS, equal 36(1) MeV and 27(1) MeV, are predicted.
6. The following masses of excited  $B_q(2S)$  states are predicted:  $M(B(2S)) = 5967$  MeV,  $M(B^*(2S)) = 6001$  MeV,  $M(B_s(2S)) = 6040$  MeV,  $M(B_s^*(2S)) = 6075$  MeV,  $M(B_c(2S)) = 6832$  MeV,  $M(B_c^*(2S)) = 6870$  MeV.

7. We predict that the mass differences for  $D^*(2S) - D(2S)$ ,  $D_s^*(2S) - D_s(2S)$  are  $\sim 72(3)$  MeV, being smaller than in several other analysis.
8. We expect that nonperturbative spin-spin potential gives not small contribution,  $\sim 15 - 30\%$  to the mass splittings for  $J/\psi - \eta_c(1S)$ ,  $D^*(1S) - D(1S)$ , and  $D_s^*(1S) - D_s(1S)$ .

For singlet states,  $D(2^1S_0)$  and  $D_s(2^1S_0)$ , one cannot exclude that they have not small hadronic shifts due to coupling to open channels; if such hadronic shifts are small, then observation of their masses, close to the values  $\sim 2.57(1)$  GeV and  $2.66(1)$  GeV, can be considered as a crucial test of a universal character of spin-spin interaction.

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- [1] G. Godfrey, and N. Isgur, Phys. Rev. D **32**, 189 (1985).
  - [2] E. J. Eichten and C. Quigg, Phys. Rev. D **49**, 5845 (1994);  
W. Buchmuller and S. H. H. Tye, Phys. Rev. D **24** (1981).
  - [3] M. DiPierro and E. J. Eichten, Phys. Rev. D **64**, 114004 (2001).
  - [4] D. Ebert, R. N. Faustov, and V. O. Galkin, Mod. Phys. Lett.A **18**, 601, 1597 (2003);  
S. Godfrey and J. L. Rosner, Phys. Rev. D **64**, 097501 (2001) and references therein.
  - [5] B. A. Kniel et al., Phys. Rev. Lett. **92**, 242001 (2001);  
S. Recksiegel and Y. Zumino, Phys. Lett. B **578**, 369 (2004);  
N. Brambilla and A. Vairo, Acta Phys. Polon. B **38**, 3429 (2007).
  - [6] A. M. Badalian and B. G. L. Bakker, Phys. Rev. D **67**, 071901 (2003).
  - [7] C. Amsler et al. (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
  - [8] S. K. Choi et al. (Belle Collab.), Phys. Rev. Lett. **89**, 142991 (2002); K. Abe et al. (Belle Collab.), Phys. Rev. D **70**, 071102(R) (2004).
  - [9] D. M. Asner et al. (CLEO Collab.), Phys. Rev. Lett. **92**, 142002 (2004).
  - [10] B. Aubert et al., (BaBar Collab.) Phys. Rev. Lett. **103**, 161801 (2009);  
B. Aubert et al., (BaBar Collab.) Phys. Rev. Lett. **101**, 071801 (2008).
  - [11] G. Bonvicini et al. (CLEO Collab.), [arXiv:0909.5474](#) [hep-ex].
  - [12] A. M. Badalian and B. G. L. Bakker, Phys. Lett.B **646**, 29 (2007);  
Phys. Atom. Nucl. **70**, 1764 (2007).
  - [13] Y. Koma and M. Koma, Nucl. Phys. B **769**, 79 (2007).
  - [14] A. De Rujula, H. Georgi, and S. L. Glashow, Phys. Rev. D **12**, 147 (1975).
  - [15] A. DiGiacomo, H. G. Dosch, V. I. Shevchenko, and Yu. A. Simonov, Phys. Rep. **372**, 319 (2002);  
H. G. Dosch and Yu. A. Simonov, Phys. Lett. B **205**, 339 (1988).
  - [16] Yu. A. Simonov, "QCD and theory of Hadrons", Lectures at XVII Int. School, Lisbon, 29 Sept.-4 Oct., 1999; [arXiv:hep-ph/9911237](#).
  - [17] A. M. Badalian, A. V. Nefediev, and Yu. A. Simonov, Phys. Rev. D **78**, 114020 (2008);  
A. M. Badalian and Yu. A. Simonov, Phys. Atom. Nucl. **59**, 2164 (1996) [Yad. Fiz. **59**, 2247 (1996)].
  - [18] A. Gray et al. (HPQCD and UKQCD Collab.), Phys. Rev. D **72**, 094507 (2005).
  - [19] T. W. Chiu et al. (TWQCD Collab.), Phys. Lett. B **651**, 171 (2007).
  - [20] B. T. Burch et al. (Fermilab Lattice and MILC Collab.) Phys. Rev. D **81**, 034508 (2010).
  - [21] B. Gregory et al. (HPQCD Collab.), Phys. Rev. Lett. **104**, 022001 (2010);  
E. Follana et al. (HPQCD and Fermilab. Lattice Collab.), Phys. Rev. Lett. **94**, 172001 (2005).
  - [22] B. Blossier et al. (ALPHA Collab.), [arXiv:1004.2661](#) [hep-lat].
  - [23] A. M. Badalian, B. G. L. Bakker, and I. V. Danilkin, Phys. Rev. D **81**, (2010).
  - [24] A. M. Badalian, B. G. L. Bakker, and V. L. Morgunov, Phys. Atom. Nucl. **63**, 1635 (2000); A. M. Badalian and V. L. Morgunov, Phys. Rev. D **60**, 116008 (1999).
  - [25] A. M. Badalian, A. I. Veselov, and B. L. G. Bakker, J. Phys. G **31**, 417 (2005); A. M. Badalian, B. L. G. Bakker, and I. V. Danilkin, Phys. Atom. Nucl. **73**, 138 (2010).
  - [26] A. M. Badalian and B. L. G. Bakker, JETP Lett. **86**, 634 (2008);  
[hep-ph/0702229](#) (2007).
  - [27] Y. Schroder, Phys. Lett. B **447**, 321 (1999);  
M. Peter, Phys. Rev. Lett. **78**, 602 (1997); Nucl. Phys. B **501**, 471 (1997).
  - [28] E. J. Eichten and C. Quigg, Phys. Rev. D **52**, 1726 (1995).
  - [29] J. Pantaleone, S. H. H. Tye, and Y. J. Ng, Phys. Rev. D **33**, 777 (1986).

- [30] A. Y. Dubin, A. B. Kaidalov, and Yu. A. Simonov, Phys. Atom. Nucl. **56**, 1745 (1993); Phys. Lett. B **323**, 41 (1994); A. V. Dubin and E. L. Gubankova, Phys. Lett. B **334**, 180 (1994).
- [31] Yu. A. Simonov, Z.Phys. C **53**, 419 (1992); Yu. S. Kalashnikova, A. V. Nefediev, and Yu. A. Simonov, Phys. Rev. D **64**, 014037 (2001); A. M. Badalian, B. L. G. Bakker, and I. V. Danilkin, Phys. Rev. D **79**, 037505 (2009); A. M. Badalian and I. V. Danilkin, Phys. Atom. Nucl. **72**, 1206 (2009).
- [32] Yu. A. Simonov, Phys. Lett. B **515**, 137 (2001).
- [33] Yu. A. Simonov and V. I. Shevchenko, Phys. Rev. Lett. **85**, 1811 (2000).
- [34] G. S. Bali, Phys. Rep. **343**, 1 (2001) (and references therein).
- [35] C. Bernard et al., (MILC Collab.) Phys. Rev. D **64**, 054506 (2001).
- [36] A. M. Badalian and D. S. Kuzmenko, Phys. Rev. D **65**, 016004 (2002); A. M. Badalian, Phys. Atom. Nucl. **63**, 2173 (2000) [Yad. Fiz. **63**, 2269 (2000)].
- [37] Yu. A. Simonov, Phys. Atom. Nucl. **65**, 135 (2002); *ibid.* **58**, 107 (1995); A. M. Badalian and Yu. A. Simonov, Phys. Atom. Nucl. **60**, 630 (1997) [Yad. Fiz. **60**, 714 (1997)].
- [38] J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982); G. Parisi and R. Petronzio, Phys. Lett. B **94**, 51 (1980); A. C. Mattingly and P. M. Stevenson, Phys. Rev. D **49**, 437 (1994).
- [39] A. M. Badalian and A. I. Veselov, Phys. Atom. Nucl. **68**, 582 (2005) [Yad. Fiz. **68**, (2005)].
- [40] A. M. Badalian, A. I. Veselov, and B. L. G. Bakker, Phys. Rev. D **70**, 016007 (2004).
- [41] Yu. A. Simonov, Phys. Atom. Nucl. **71**, 1048 (2008); *ibid.* **68**, 709 (2005).
- [42] A. M. Badalian and I. V. Danilkin, Phys. Atom. Nucl. **72**, 1206 (2009); J. L. Rosner, Phys. Rev. D **64**, 094001 (2001); Y. P. Kuang and T. M. Yan, Phys. Rev. D **41**, 155 (1990).
- [43] Yu. A. Simonov and V. I. Shevchenko, [arXiv:0902.1405](#); Yu. A. Simonov, Phys. Atom. Nucl. **69**, 528 (2006).
- [44] A. M. Badalian, B. L. G. Bakker, and Yu. A. Simonov, Phys. Rev. D **66**, 034026 (2002).
- [45] S. F. Radford, W. W. Repko, and M. J. Saelim, Phys. Rev. D **80**, 034012 (2009).
- [46] T. Matsuki, T. Morio, and K. Sudoh, Eur. Phys. J. A **31**, 701 (2007), [hep-ph/0610186](#).
- [47] B. Aubert et al. (BaBar Collab.), Phys. Rev. D **80**, 071103 (2009); *ibid.* Phys. Rev. Lett. **97**, 222001 (2006); J. Brodzicka et al. (Belle Collab.), Phys. Rev. Lett. **100**, 092001 (2008).
- [48] A. M. Badalian, B. L. G. Bakker, and Yu. A. Simonov, Phys. Rev. D **75**, 116001 (2007).
- [49] A. DiGiacomo and Yu. A. Simonov, Phys. Lett. B **595**, 368 (2004).

## VIII. APPENDIX A. THE POLE MASS OF A HEAVY QUARK

The RSH (7) contains the pole mass of a heavy quark  $m_2 = m_Q$  and of a lighter quark  $m_1$ . For a light quark,  $m_u$  or  $m_d$ , its mass is taken equal 5 MeV, while for a  $s$  quark  $m_s = 200$  MeV is used as in [26]. The pole masses of heavy quarks ( $c, b$ ) are defined as in pQCD, when the pole mass is expressed via the QCD current mass  $\bar{m}_Q(m_Q)$ , entering the QCD Lagrangian, and higher order corrections of the strong coupling  $\alpha_s$  [7]:

$$m_Q(pole) = \bar{m}_Q(m_Q) \left[ 1 + \frac{4\alpha_s(\bar{m}_Q)}{3\pi} + r_2 \left( \frac{\alpha_s}{\pi} \right)^2 \right], \quad (\text{A.1})$$

where the factor  $r_2$ ,

$$r_2(n_f) = 13.4434 - 1.0414 \sum_{k=1}^{N_L} \left( 1 - \frac{4\bar{m}_{Q_k}}{3\bar{m}_Q} \right), \quad (\text{A.2})$$

depends on a number of flavors  $n_f$  through the sum, which goes from  $k = 1$  up to  $N_L = n_f - 1$ . Due to this term the pole mass appears to be slightly different for different  $n_f$ . Here in our calculations we take  $m_b(n_f = 5) = 4.823(3)$  GeV and  $m_b(n_f = 4) = 4.79$  GeV; a small difference between them lies within a theoretical error, present in the current mass,  $m_b(\bar{m}_b) = 4.20 \pm 0.07$  GeV [7]. The  $m_b(pole)$  used here correspond to the value of the current mass  $\bar{m}_b(\bar{m}_b) = 4.210 \pm 0.015$  GeV, which is within the conventional number [7].

For a  $c$  quark the pole mass  $m_c(n_f = 4) = 1.41$  GeV is used for charmonium, and the  $B_c, D_s$  mesons; it corresponds to the conventional value,  $\bar{m}_c = 1.24 \pm 0.09$  GeV [7]. For the  $D$  mesons  $m_c(pole) = 1.39$  GeV is used.

## IX. APPENDIX B. THE QUARK SELF-ENERGY CONTRIBUTION TO A MESON MASS

The nonperturbative quark self-energy (SE) contribution  $\Delta_{SE}$  to a meson mass  $M(nL)$  was calculated in FCM [32], where the meson Green's function was defined in a gauge-invariant way. This correction,

$$\Delta_{SE} = -\frac{1.5\sigma\eta(q)}{\pi\omega_q} \quad (\text{A.3})$$

is negative, proportional to the string tension  $\sigma$ , and a quark kinetic energy as  $\omega_q^{-1}$ , which is rather large for a light quark and small for a heavy quark. In (A.3) the factor  $\eta_q$  (a number) depends on a quark mass:  $\eta_n = 1.0$  for a light quark;  $\eta_s \simeq 0.80$  for a  $s$  quark;  $\eta_c \simeq 0.40$  for a  $c$  quark, and  $\eta_b \simeq 0.2$  for a  $b$  quark [48], [32]. Notice that the SE term (A.3) contains correct coefficient 1.5, instead of the coefficient 2.0 in [32]; the reasons for a change of this number is discussed in [49].

For a  $b$ -quark the SE contribution is small ( $\leq (-1)$  MeV) and can be neglected; for a  $c$  quark its value is  $\sim (-20)$  MeV and it is convenient to include this small correction via a redefinition of a pole mass. For the ground states of heavy-light mesons the SE contribution, which comes from a light, is rather large, being  $\sim (-140)$  MeV, and a bit smaller,  $\sim (-90)$  MeV, for a  $s$  quark. For higher states of heavy-light mesons the SE contributions are smaller, because for them the kinetic energy of a light ( $s$  quark), present in (A.3), is larger.